1. A DFA OVER A FINITE ALPHABET Σ CAN BE EXPRESSED VIA AN EDGE-LABELED DIGRAPH AS FOLLOWS:

   (a) EACH VERTEX OF THE GRAPHS REPRESENTS A STATE OF THE AUTOMATON.

   (b) EACH STATE HAS ONE OUTGOING EDGE FOR EACH SYMBOL OF Σ, INDICATING ITS TRANSITIONS.

   (c) THE START STATE IS INDICATED BY AN "INCOMING EDGE" COMING FROM NOWHERE.

   (d) EACH STATE CAN BE EITHER MARKED AS AN ACCEPT STATE (INDICATED BY CIRCLING IT) OR NOT — THE UNMARKED STATES ARE REJECT STATES.

   (e) GIVEN AN INPUT STRING $x \in \Sigma^*$, A DFA OPERATES AS FOLLOWS:

   - START AT THE START STATE.
   - FOR EACH CHARACTER OF x IN TURN, FOLLOW THE CORRESPONDING TRANSITION TO MOVE TO SOME NEXT STATE (POSSIBLY COMING BACK TO THE STATE YOU'RE AT, IN THE CASE OF A LOOP EDGE).
   - AFTER PROCESSING ALL CHARACTERS OF x:
     - IF THE CURRENT STATE IS AN ACCEPT STATE, THE DFA ACCEPTS THE STRING x.
     - OTHERWISE, THE DFA REJECTS THE STRING x.

2. A DFA TAKES AS INPUT A STRING $x \in \Sigma^*$, AND FOR EACH ONE, IT RETURNS EITHER ACCEPT OR REJECT. THUS, A DFA "M" OVER Σ ACTS LIKE A FUNCTION $M: \underbrace{\Sigma^*}_{\text{DOMAIN}} \to \underbrace{\{\text{ACCEPT}, \text{REJECT}\}}_{\text{CODOMAIN}}$.

3. EACH STRING $x \in \Sigma^*$ HAS US START AT THE START STATE AND STOP AFTER A FINITE NUMBER OF STEPS (BECAUSE EACH $x \in \Sigma^*$ HAS FINITE LENGTH). THUS, WE MUST FINISH AT SOME STATE, WHICH WILL EITHER BE AN ACCEPT STATE OR NOT.

   (THIS MIGHT SEEM LIKE A SILLY QUESTION, AND IT IS A LITTLE SILLY FOR DFA'S... BUT OTHER TYPES OF MACHINES — INCLUDING REGULAR COMPUTERS RUNNING PROGRAMS — CAN GET STUCK IN AN INFINITE LOOP AND NOT FINISH. NOT DFA'S, THOUGH!)

   WE DEFINE THE LANGUAGE ACCEPTED BY A DFA "M" OVER Σ AS THE SET OF ALL STRINGS THAT M ACCEPTS:

$$L(M) = \{x \in \Sigma^* : M \text{ ACCEPTS } x\}$$

4. EACH DFA M GIVES US A SUBSET $L(M) \subset \Sigma^*$ ACCEPTED BY M. I.E., AN ELEMENT OF $\mathcal{P}(\Sigma^*)$.

   WE CAN THINK OF THIS ABSTRACTLY A FUNCTION

$$\begin{aligned} f: \{M : M \text{ IS A DFA OVER } \Sigma\} &\longrightarrow \mathcal{P}(\Sigma^*) \\ M &\longmapsto L(M) \end{aligned}$$

5. (a)(i) A ACCEPTS ANY STRING STARTING WITH 101. 101 GETS TO THE BOTTOM (ACCEPT) STATE, AND ALL SUBSEQUENT CHARACTERS RETURN US THERE ⇝ ACCEPT

(ii) A REJECTS ANY STRING STARTING WITH 100. THE SECOND 0 SENDS US TO THE STATE AT THE LEFT, WHERE WE'LL STAY ⇝ REJECT.

(iii) A REJECTS ANY STRING STARTING WITH 11. THE SECOND 1 SENDS US TO THE STATE AT THE LEFT, WHERE WE'LL STAY ⇝ REJECT

(iv) A REJECTS ANY STRING STARTING WITH 0. THE 0 SENDS US TO THE STATE AT THE LEFT, WHERE WE'LL STAY ⇝ REJECT

(b) A ACCEPTS PRECISELY THE STRINGS STARTING WITH 101, I.E., $\underbrace{101(0+1)^*}_{L(A)}$

6. (a) (i) B REJECTS THE STRING ε. IT STARTS AT THE START STATE & NEVER MOVES ⇝ REJECT

(ii) B ACCEPTS THE STRING 1000100. IT STAYS AT THE START UNTIL THE 000, WHICH GETS IT TO THE ACCEPT STATE, WHERE IT STAYS ⇝ ACCEPT

(iii) B REJECTS THE STRING 00100100. THE 00'S GET US TO THE BOTTOM-RIGHT STATE, BUT WITHOUT A THIRD 0, WE ARE SENT BACK TO START ⇝ REJECT

(iv) B ACCEPTS THE STRING 11100011. JUST LIKE SUBPART (ii) — THE 000 GETS US TO THE ACCEPT STATE.

(b) B ACCEPTS PRECISELY THE STRINGS CONTAINING 000, I.E., $\underbrace{(0+1)^*000(0+1)^*}_{L(B)}$

7. (a) (i) C ACCEPTS THE STRING ε. WE START AT THE START STATE & DON'T MOVE; THE START STATE IS AN ACCEPT STATE ⇝ ACCEPT

(ii) C ACCEPTS THE STRING 001. THE 001 SENDS US ALL THE WAY AROUND THE TRIANGLE, BACK TO THE ACCEPT STATE ⇝ ACCEPT

(iii) C ACCEPTS THE STRING 001001. JUST LIKE SUBPART (ii), BUT TWICE AROUND THE TRIANGLE ⇝ ACCEPT

(iv) C REJECTS THE STRING 00100100. AFTER THE SECOND 1, WE'RE AT THE ACCEPT STATE; BUT THE TWO 0'S AT THE END LEAVE US AT THE BOTTOM-RIGHT ⇝ REJECT

(b) C ACCEPTS PRECISELY THE NONNEGATIVE POWERS OF 001, I.E., $\underbrace{(001)^*}_{L(C)}$

8. (a) (i) D ACCEPTS THE STRING ε. WE START AT THE START STATE & DON'T MOVE; THE START STATE IS AN ACCEPT STATE ⇝ ACCEPT

(ii) D ACCEPTS THE STRING 1111. THE FIRST 1 MOVES US TO THE LOWER ACCEPT STATE, AND ALL SUBSEQUENT 1'S KEEP US THERE ⇝ ACCEPT

(iii) D ACCEPTS THE STRING 00011. THE 0'S KEEP US AT THE START STATE, THEN THE 1'S DO AS IN SUBPART (ii) ⇝ ACCEPT

(iv) D REJECTS THE STRING 0011100. THE START 0011 LANDS US AT THE LOWER START STATE, JUST AS IN SUBPART (iii); BUT THE SUBSEQUENT 0 SENDS US TO THE BOTTOM STATE, WHERE WE STAY ⇝ REJECT

(b) D ACCEPTS PRECISELY THE STRINGS THAT HAVE NO 0'S AFTER 1'S, I.E., SOME NONNEGATIVE NUMBER OF 0'S FOLLOWED BY SOME NONNEGATIVE NUMBER OF 1'S: $\underbrace{0^*1^*}_{L(D)}$

INITIAL 0'S KEEP US AT THE START (ACCEPT) STATE; ANY 1 SENDS US TO THE LOWER ACCEPT STATE, WHERE WE STAY UNLESS WE SEE A 0 — A 0 AFTER A 1 SENDS US TO THE REJECT STATE, WHERE WE STAY.

* NOTE: THESE ARE EXACTLY THE REGULAR EXPRESSIONS FROM THE PREVIOUS PROBLEM SET IN PROBLEM 7!

REASONABLE QUESTIONS:
- FOR EVERY DFA M, IS L(M) GIVEN BY A REGULAR EXPRESSION?
- DOES EVERY REGULAR EXPRESSION GIVE L(M) FOR SOME DFA M?

(ANSWERS: YES AND YES!)